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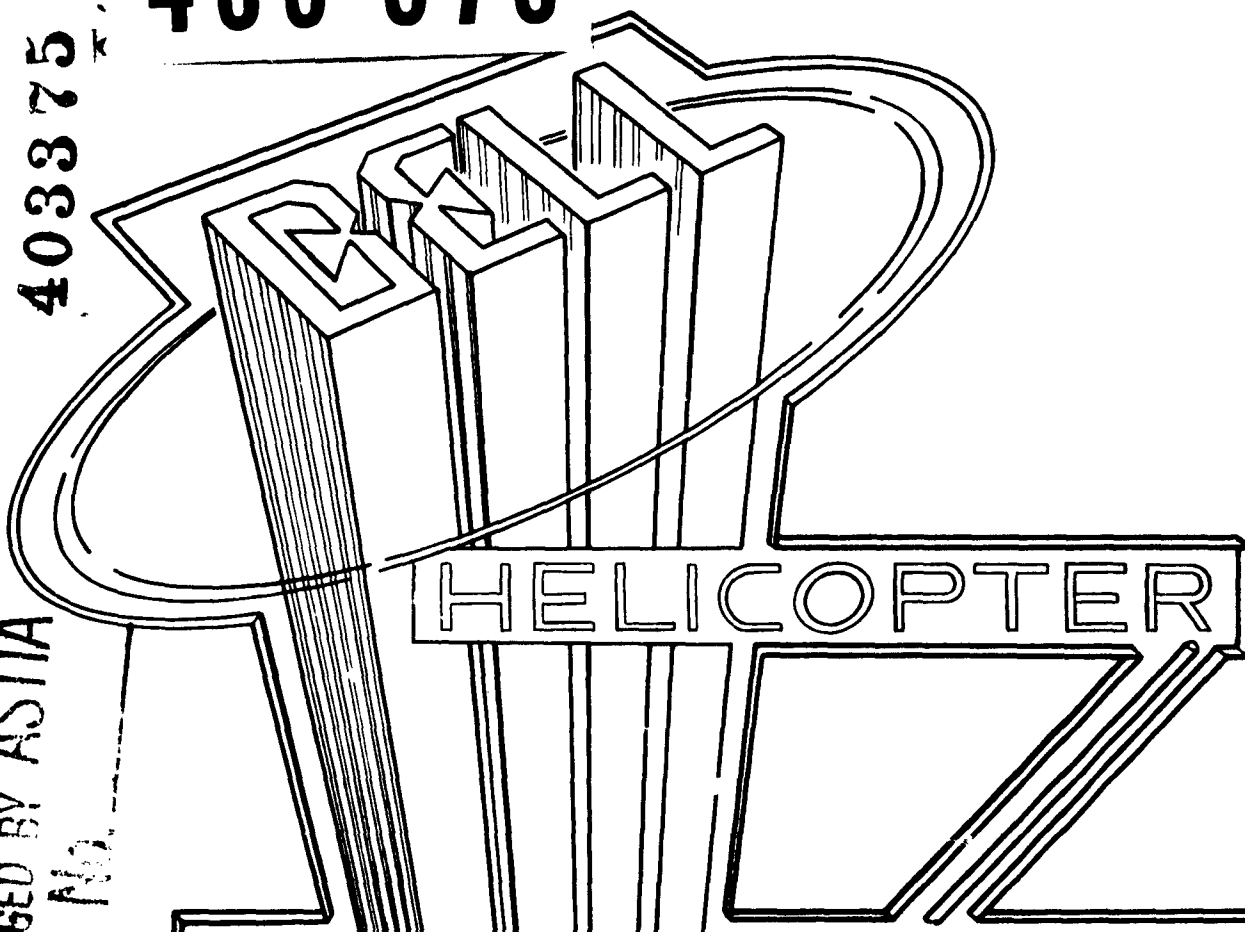
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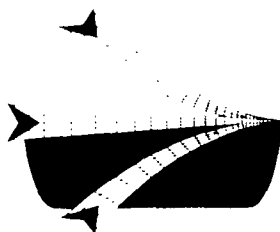
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**ANIP**

The derivation of a  
coupling network for  
the dynamic simulator  
platform.

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THE DERIVATION OF A COUPLING NETWORK  
FOR THE DYNAMIC SIMULATOR PLATFORM

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### SUMMARY

An interim solution to the problem of scaling dynamic simulator platform motions is presented in this report which derives the filter network between the airframe computer and the platform servo system to give minimum acceleration error subject to the constraint that the platform motion is confined to limited values.

The constants of this optimal coupling filter are given as a function of the expected rms velocity for the various degrees of freedom of the system.

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THE DERIVATION OF A COUPLING NETWORK  
FOR THE DYNAMIC SIMULATOR PLATFORM

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I. INTRODUCTION

A discussion of the general problem of scaling of platform motions to a particular vehicular system has been made in Report No. D228-430-005. This report is an extension of D228-430-005 in which a discussion is given of the portion of the problem dealing with the development of a filtering network to put between the airframe computer and the servo system of the dynamic simulator platform. The need for some filtering of the signals from the airframe computer arises primarily from the necessity to simulate what amounts to unlimited motions of the aircraft in some axes with the limited motions of the dynamic platform. As an example, in vertical flight the helicopter is capable of attaining altitudes of several thousand feet, whereas the platform on which we intend to simulate these motions has a total vertical travel of 13 feet. The mapping of these large excursions into limited motion of the platform may be accomplished a number of ways. The criteria for accomplishment, however, is the faithful reproduction of the vestibular and kinesthetic cues which the pilot actually is capable of detecting as he operates the machine. Eventually we hope to be able to determine the thresholds of feeling in the various axes of motion and, using these data, calculate a filtering network to put between the computer and the platform.

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This filter will then supply those motion cues which can be felt and suppress those which cannot so that the motion of the platform will, in fact, faithfully reproduce the necessary cues while staying within the confines of its motion.

To date, threshold data for the human operator have not been sufficiently defined to allow the complete analysis of the problem. This is principally due to limitations in time and certain features of the platform which make the measurement of motion thresholds difficult. At some future time we hope to have enough data to complete the design of the filter system on a more rigorous basis; however, at the present the need for a filter network becomes more and more pressing as the time for the first experiment approaches.

This report is intended to describe an interim solution to the problem which makes maximum use of the available platform capabilities subject to the assumption that the acceleration reproduction is most important. The method outlined in this report is based on the calculus of variations in which we derive a filter circuit which produces the maximum acceleration possible within the confines of the platform movement. Certain assumptions have been made, and in part these are justified by such experimental evidence as we can collect at this time. Principally the assumptions are that the inputs to the control system by the pilot constitute a random motion and that the aircraft response may be represented as a simple filter whose characteristic frequency is determined from the equations of motion of the aircraft.

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## II. FORMULATION OF THE PROBLEM

In order to derive a reasonable form for the input to the platform the assumption is made that the stick motions made by the pilot constitute a white random noise, that is, one whose spectral density is constant and extends from zero to infinite frequency. This assumption, although unrealistic since we know that human response falls off at higher frequencies, provides an upper limit to the stick inputs and hence represents a situation which is actually somewhat pessimistic. This input is then filtered through the aircraft transfer functions to produce motion signals for the platform. In this analysis we assume that the transfer functions are specified and, in fact, will use a simplified transfer function which utilizes the lowest natural frequency of the system only. This assumption is made not from necessity of the method, but rather a simplification of the calculations to be carried out. In this way we can arrive at a power spectral density function for the inputs to the platform. These may be characterized by parameters which are available or relatively easy to measure.

The methods of variational calculus which are used in this analysis allow us to minimize some error function subject to the constraint that some other variable be held constant. With respect to the problem of filtering of the platform signals we will choose the error as being the difference between the acceleration of the aircraft as determined by the output of the airframe computer and the acceleration of the platform. The constraint which we can use is the limitation of



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platform movement in the various axes. Because of the nature of the problem and the method of solution, we must accept a probabilistic argument for both the error and the constraint, that is to say we can only define the variance of the error and hold the variance of the position to within certain limits. Physically, this will mean that the platform can at times be commanded to exceed its limits of travel and hence run into the stops. However, if we choose the constraints properly we may control the probability that this happens to any desired value. Figure 1 shows a block diagram of the system. Because the characteristic frequencies of the aircraft are low we may expect that the platform will be commanded to move principally within the frequency range over which its transfer function is essentially one, that is, at frequencies below one cycle per second. This will allow us to make the assumption that the platform transfer function is one. Combining these assumptions, we may then draw a diagram as in Figure 2, in which the computed aircraft position is fed to the filter with impulse response  $W(t)$ , giving the platform output position  $X_p(t)$ . The error to be minimized is the difference in acceleration between the aircraft and the platform. This circuit is representative of the problem to be solved in a single axis. Although there is cross-coupling between axes we have assumed independence for this analysis. The general solution for the weighting function of the filter may be carried out and then numerical values substituted for each of the four degrees of freedom in which the platform is to be excited.

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Figure 1.

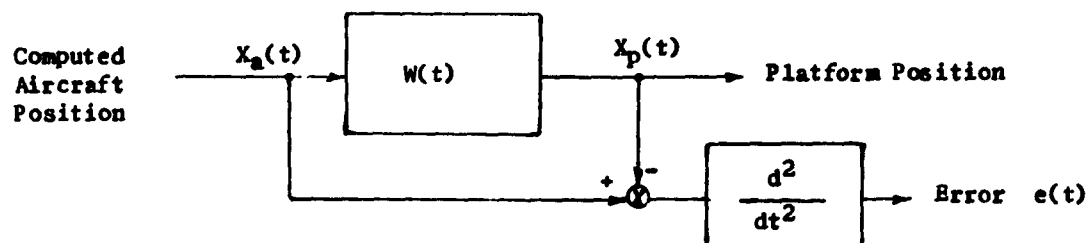


Figure 2.

### III. DERIVATION OF THE OPTIMAL FILTER

The problem of selecting the optimal function  $W(t)$  subject to the conditions that the mean square value of  $e(t)$  be a minimum and the mean square value of  $X_p(t)$  be less than or equal to some predetermined value is a model of the problem of selecting the platform filtering necessary. In order to solve this problem we will use the method of calculus of variations in which a functional equation is set up as follows:

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$$F = \overline{e^2} + \lambda \overline{x_p^2} \quad (1)$$

Where  $F$  represents the functional to be minimized

$\overline{e^2}$  is the mean square error as defined above

$\overline{x_p^2}$  is the mean square value of the platform displacement

$\lambda$  is the Lagrange multiplier

Since we wish to minimize the functional  $F$  with respect to the impulse response,  $W(t)$ , of the filter the mean square error  $\overline{e^2}$  and the mean square displacement  $\overline{x_p^2}$  must be expressed in terms of  $W(t)$ . In order to express  $x_p(t)$  in terms of the input,  $x_a(t)$ , and the impulse response,  $W(t)$ , we can use the convolution integral relationship between the input and output of a linear network.

Thus,

$$x_p(t) = \int_{-\infty}^{\infty} W(t_1) x_a(t-t_1) dt_1 \quad (2)$$

so that

$$x_p^2(t) = \int_{-\infty}^{\infty} dt_1 \int_{-\infty}^{\infty} dt_2 W(t_1) W(t_2) x_a(t-t_1) x_a(t-t_2) \quad (3)$$

where  $t_1$  and  $t_2$  have been used to avoid confusion of the variable of integration.

To determine the mean square value of  $x_p$  we utilize the definition

$$\overline{x_p^2} \triangleq \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T x_p^2(t) dt \quad (4)$$

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or

$$\overline{X_p}^2 = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T dt \int_{-\infty}^{\infty} dt_1 \int_{-\infty}^{\infty} dt_2 W(t_1) W(t_2) X_a(t-t_1) X_a(t-t_2) \quad (5)$$

We have assumed that  $X_a(t)$  is a random variable so that the averaging process operates on the product  $X_a(t-t_1)X_a(t-t_2)$ . Therefore, we may exchange the order of integration, giving

$$\overline{X_p}^2 = \int_{-\infty}^{\infty} dt_1 \int_{-\infty}^{\infty} dt_2 W(t_1) W(t_2) \left\{ \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T X_a(t-t_1) X_a(t-t_2) dt \right\} \quad (6)$$

The expression

$$\lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T X_a(t-t_1) X_a(t-t_2) dt$$

is by definition the autocorrelation function,  $\vartheta_{aa}(t_1-t_2)$ , of the variable  $X_a(t)$ . We have assumed implicitly here that  $X_a(t)$  is a stationary random process.

Therefore,

$$\overline{X_p}^2 = \int_{-\infty}^{\infty} dt_1 \int_{-\infty}^{\infty} dt_2 W(t_1) W(t_2) \vartheta_{aa}(t_1-t_2). \quad (7)$$

To determine the relationship between the mean square error,  $\overline{e}^2$ , and the impulse response  $W(t)$  we can go through a similar argument resulting in the expression,

$$\begin{aligned} \overline{e}^2 &= \int_{-\infty}^{\infty} dt_2 \int_{-\infty}^{\infty} dt_4 h(t_2) h(t_4) \vartheta_{aa}(t_2-t_4) \\ &\quad - 2 \int_{-\infty}^{\infty} dt_1 \int_{-\infty}^{\infty} dt_2 \int_{-\infty}^{\infty} dt_4 h(t_2) h(t_4) W(t_1) \vartheta_{aa}(t_1+t_2-t_4) \\ &\quad + \int_{-\infty}^{\infty} dt_1 \int_{-\infty}^{\infty} dt_2 \int_{-\infty}^{\infty} dt_3 \int_{-\infty}^{\infty} dt_4 h(t_2) h(t_4) W(t_1) W(t_3) \vartheta_{aa}(t_1+t_2-t_3-t_4) \end{aligned} \quad (8)$$

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where  $h(t)$  is the impulse response corresponding to the operation  $\frac{d^2}{dt^2}$ .

The functional  $F$  may now be formed as a function of the unknown,  $W(t)$ . To solve for this unknown we must introduce an arbitrary variation in the function  $W(t)$  and minimize  $F$  with respect to this variation.

To introduce the variation in  $W(t)$ , let

$$W(t) = W_0(t) + \epsilon W_\epsilon(t) \quad (9)$$

$W_0(t)$  = optimal weighting function

$W_\epsilon(t)$  = arbitrary variation of  $W(t)$

$\epsilon$  = system (or equations) degree of freedom.

We now have  $F$  as a function of  $\epsilon$  so that to solve for  $W_0(t)$ , the optimal weighting function, we may let

$$\left. \frac{\partial F}{\partial \epsilon} \right|_{\epsilon=0} = 0. \quad (10)$$

To form the functional equation  $F$ , substitute equations (7) and (8) into (1).

$$\begin{aligned}
 F = & \int_{-\infty}^{\infty} dt_2 \int_{-\infty}^{\infty} dt_4 h(t_2)h(t_4)g_{aa}(t_2-t_4) \\
 & - 2 \int_{-\infty}^{\infty} dt_1 \int_{-\infty}^{\infty} dt_2 \int_{-\infty}^{\infty} dt_4 h(t_2)h(t_4)W(t_1)g_{aa}(t_1+t_2-t_4) \\
 & + \int_{-\infty}^{\infty} dt_1 \int_{-\infty}^{\infty} dt_2 \int_{-\infty}^{\infty} dt_3 \int_{-\infty}^{\infty} dt_4 h(t_2)h(t_4)W(t_1)W(t_3)g_{aa}(t_1+t_2-t_3-t_4) \\
 & + \lambda \int_{-\infty}^{\infty} dt_1 \int_{-\infty}^{\infty} dt_2 W(t_1)W(t_2)g_{aa}(t_1-t_2), \quad (11)
 \end{aligned}$$

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Substituting equation (9) into (11) and carrying out the operations indicated by equation (10), we have

$$0 = \int_{-\infty}^{\infty} dt_1 W(t_1) \left\{ \int_{-\infty}^{\infty} dt_2 \int_{-\infty}^{\infty} dt_3 \int_{-\infty}^{\infty} dt_4 h(t_2) h(t_4) W_0(t_3) g_{aa}(t_1 + t_2 - t_3 - t_4) \right. \\ \left. + \lambda \int_{-\infty}^{\infty} dt_2 W_0(t_2) g_{aa}(t_1 - t_2) - \int_{-\infty}^{\infty} dt_2 \int_{-\infty}^{\infty} dt_4 h(t_2) h(t_4) g_{aa}(t_1 + t_2 - t_4) \right\} \quad (12)$$

Since  $W(t_1)$  is arbitrary it is necessary that the expression within the brackets be zero, therefore

$$\int_{-\infty}^{\infty} dt_2 \int_{-\infty}^{\infty} dt_3 \int_{-\infty}^{\infty} dt_4 h(t_2) h(t_4) W_0(t_3) g_{aa}(t_1 + t_2 - t_3 - t_4) \quad (13) \\ + \lambda \int_{-\infty}^{\infty} dt_2 W_0(t_2) g_{aa}(t_1 - t_2) = \int_{-\infty}^{\infty} dt_2 \int_{-\infty}^{\infty} dt_4 h(t_2) h(t_4) g_{aa}(t_1 + t_2 - t_4).$$

Because we wish the function  $W_0(t)$  to be physically realizable (i.e.,  $W(t) = 0$ ;  $t < 0$ ) we impose the further restriction on equation (13) that it applies only for  $t_1 > 0$ . Under these conditions eq (13) becomes a Weiner-Hopf integral equation of the first kind which may be solved by methods of transformation, either Laplace or Fourier, and separation of the resulting complex expressions. The explanation of this method of solution is given very clearly in reference 1, and will not be repeated here. The essential results of the method are utilized to solve eq (13) for the Laplace transform of the impulse response.

Let us define:

$$L [f(t)] = F(s) \quad (14)$$

Where  $L [ \quad ]$  denotes the Laplace transform of the function within the brackets.

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Taking the Laplace transform of both sides of eq (13) we have

$$W_0(s) \left\{ H(s) \cdot H(-s) + \lambda \right\} G_{aa}(s) = H(s) \cdot H(-s) G_{aa}(s) \quad (15)$$

Where

$$H(s) = L [h(t)]$$

$$H(-s) = \text{Complex conjugate of } H(s)$$

$$G_{aa}(s) = L [g_{aa}(t)] = \text{Power spectrum of } X_a(t).$$

The method of solution of the Weiner-Hopf equation for a physically realizable  $W_0(s)$  is summarized by:

$$W_0(s) = \frac{\left[ \frac{H(s) \cdot H(-s) G_{aa}(s)}{[(H(s) \cdot H(-s) + \lambda) G_{aa}(s)]_-} \right]_+}{[(H(s) \cdot H(-s) + \lambda) G_{aa}(s)]_+}, \quad (16)$$

Where

$[F(s)]_-$  denotes the portion of  $F(s)$  having poles and zeros in the right half of the  $s$  plane.

$[F(s)]_+$  denotes the portion of  $F(s)$  having poles and zeros in the left half of the  $s$  plane.

Equation (16) may be further simplified by noting that the function  $G_{aa}(s)_+$  appears in both numerator and denominator of the expression for  $W_0(s)$ ; therefore,

$$W_0(s) = \frac{H(s)}{[H(s) \cdot H(-s) + \lambda]_+}. \quad (17)$$

The expression given in eq (17) represents the general solution of the problem for the optimal weighting function  $W_0(t)$  in terms of its Laplace transform. The problem of evaluating the Lagrange multiplier,  $\lambda$ , still

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remains to be solved. The transfer function of the optimal filter,  $W_0(s)$ , is given in terms of  $\lambda$ , and therefore may be used to evaluate  $\lambda$ .

We have requested that  $W_0(s)$  result in the minimum error, as weighted by  $H(s)$ , with a fixed constraint on the mean square value of  $x_p$ , the displacement of the platform.

Utilizing the results of Generalized Harmonic Analysis (see Ref.2) we may express  $\overline{x_p^2}$  in terms of  $W_0(s)$  by

$$\overline{x_p^2} = \int_{-\infty}^{\infty} W_0(s) \cdot W_0(-s) G_{aa}(s) ds \quad (18)$$

Since  $\overline{x_p^2}$  is fixed by the limits of the platform motion, we may carry out the integration and solve the resulting equation for  $\lambda$ .

To determine the explicit relationship between  $\lambda$  and  $\overline{x_p^2}$  we may utilize equation (17), remembering that  $H(s)$  corresponds to the operation  $\frac{d^2}{dt^2}$ .

Therefore

$$H(s) = s^2 \quad (19)$$

so that

$$W_0(s) = \frac{s^2}{[s^4 + \lambda]_+} \quad (20)$$

To determine the portion of the denominator having zeros in the left half of the  $s$  plane we note that

$$s^4 + \lambda = (s^2 + 2as + 2a^2)(s^2 - 2as + 2a^2) \quad (21)$$

giving

$$[s^4 + \lambda]_+ = s^2 + 2as + 2a^2 \quad (22)$$



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$$\text{where } a = \frac{\lambda^{\frac{1}{2}}}{\sqrt{2}}.$$

This gives  $W_O(s)$  as,

$$W_O(s) = \frac{s^2}{s^2 + 2as + 2a^2} \quad (23)$$

#### IV. CALCULATION OF NUMERICAL VALUES FOR THE OPTIMAL FILTER

To determine the numerical value of  $a$ , and hence  $\lambda$ , we must establish the form of the input spectrum  $G_{aa}(s)$ .

To determine  $G_{aa}(s)$  we must consider each of the axes of motion of the aircraft separately.

##### Case 1: Vertical Motion; Forward Flight

Using a simplified set of equations relating control movements to aircraft movements we find that for the uncoupled case the control to vertical velocity transfer function may be approximated by

$$\frac{V(s)}{b(s)} = \frac{K}{s + w_1} \quad (24)$$

where  $V(s)$  is the vertical velocity

$b(s)$  is the cyclic input

$K$  is the gain of the system

$w_1$  is the lowest characteristic frequency of the system.

In the case of vertical motion, the  $X_a(t)$  referred to in Figure 2 is the altitude of the aircraft so that to obtain the altitude response we must integrate the vertical velocity. The transfer function given in (24) then becomes

$$\frac{Z(s)}{b(s)} = \frac{K}{s(s + w_1)} \quad (25)$$

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where  $Z(s)$  is the altitude variable.

The problem of estimating the power spectrum (or the autocorrelation function) of the altitude from the information we have at hand may be best attacked by utilizing some parameter such as the mean square vertical velocity,  $\sigma_{va}^2$ , which we may estimate from physical considerations. If we further assume that the input to the controls is a white noise (having a constant spectrum for all frequencies) of unit amplitude, then the M.S. vertical velocity is given by

$$\sigma_{va}^2 = \int_0^{\infty} \frac{K w_1}{\pi (-s^2 + w_1^2)} ds \quad (26)$$

So that if we estimate  $\sigma_{va}^2$  and assume that the transfer function is given by eq (24) the power spectrum of the vertical velocity will be

$$G_{vv}(s) = \frac{\sigma_{va}^2 w_1}{\pi (-s^2 + w_1^2)} \quad (27)$$

The altitude spectrum is then available by the methods of Generalized Harmonic Analysis as,

$$G_{aa}(s) = \frac{\sigma_{va}^2 w_1}{\pi (-s^2)(-s^2 + w_1^2)} \quad (28)$$

Substituting eqs (23) and (28) into eq (18), we have for the mean square platform displacement

$$\overline{X_p^2} = \int_0^{\infty} \frac{\sigma_{va}^2 w_1}{\pi} \left| \frac{s^2}{(s)(s + w_1)(s^2 + 2as + 2a^2)} \right|^2 ds \quad (29)$$

$$\text{where } |F(s)|^2 = F(s) \cdot F(-s) \quad .$$

Since  $\bar{X}_p^2$ ,  $\sigma_{va}^2$ , and  $w_1$  are fixed for any particular aircraft, flight mode and limit of platform motion, we may let

$$k^2 = \frac{\pi \bar{X}_p^2}{\sigma_{va}^2 w_1} \quad (30)$$

so that

$$k^2 = \left| \frac{s}{(s + w_1)(s^2 + 2as + 2a^2)} \right|^2 ds. \quad (31)$$

Reference 2 gives a table of integrals of the form of eq (31) so that we find

$$k^2 = \frac{1}{4a(w_1^2 + 2a w_1 + 2a^2)}. \quad (32)$$

Since we are interested in solving for  $a$  to determine the numerical values of the filter transfer function we may rewrite eq (32) as

$$a^3 + w_1 a^2 + \frac{w_1^2 a}{2} - \frac{1}{8k^2} = 0. \quad (33)$$

In order to simplify the calculations of  $a$ , divide eq (33) by  $w_1^3$ , giving

$$\left(\frac{a}{w_1}\right)^3 + \left(\frac{a}{w_1}\right)^2 + \frac{1}{2} \left(\frac{a}{w_1}\right) - \frac{1}{8k^2 w_1^3} = 0. \quad (34)$$

Figure 3 shows a plot of the cutoff frequency  $\sqrt{2a}$  versus the rms vertical velocity with  $\bar{X}_p^2$  and  $w_1$  fixed. The values picked for  $\bar{X}_p^2$  and  $w_1$  were chosen to reflect the heave channel limitation of  $\pm 3$  feet and the H-40 characteristic frequency of 0.2512 rad/sec.

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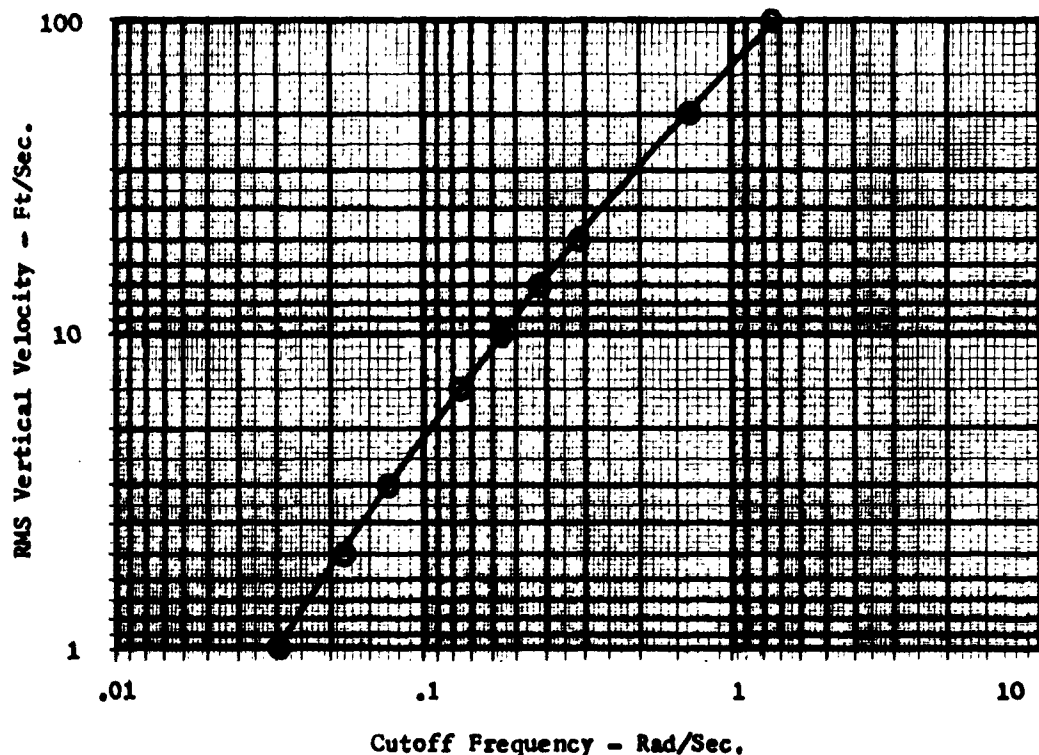
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FIGURE 3. PLOT OF FILTER CUTOFF FREQUENCY  
 $\sqrt{2a}$  VERSUS RMS VERTICAL VELOCITY.

For  $w_j = 0.2512$  R/S

$x_p = 3$  Ft.

Case 2. Rotational Motions; Forward Flight

The same general argument may be applied to rotational movements (heading or yaw changes) as to the vertical motions. The exact values of the parameters will differ due to dynamic differences in the aircraft response; however, no basic change in approach is required. Calculations of the filter parameters will be delayed until further experience and experimentation is accumulated relative to the vertical motions.

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Because the transfer functions relating pitch and roll to control inputs are of a non-integrating nature it would appear that there is no requirement for filtering the outputs of the airframe computer to the roll and pitch inputs of the platform. If problems are encountered in exceeding the limits of platform motion in these axes, further examination of the problem will be necessary.

#### V. CONCLUSIONS

The derivation of the optimal filter network which reproduces acceleration with a minimum of error while staying within the limits of platform motion gave

$$W_o(s) = \frac{s^2}{s^2 + 2as + 2a^2}$$

as the transfer function of the filter. The numerical values of the cutoff frequency,  $\sqrt{2a}$ , are given in figure 3 for a range of conditions of rms vertical velocity. The problems associated with the threshold of feeling of the human operator have been circumvented temporarily until more experimental data are available. Since the coupling computer between the airframe computer and the platform presently contains only one operational amplifier per channel a slight simplification is recommended in the filter transfer function to reduce the number of components in the filter. This simplification is justifiable since only a slight change in response results and a great deal of latitude exists in the assumptions which have been made.

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One further point must be brought out, that is, the output of the computer (airframe) is intended to be rate of change of position rather than position so the transfer function of the simplified filter to be used is

$$W_O(s) = \frac{s}{(s + \sqrt{2a})^2} \cdot$$

Some discussion of the physical significance of the three parameters,  $\sigma_{va}^2$ ,  $\bar{x}_p^2$  and  $w_1$ , which in effect control the cutoff frequency,  $\sqrt{2a}$ , is in order to demonstrate the flexibility of the derivation of the filter. If the subject is asked to maintain level flight,  $\sigma_{va}^2$ , may be considered as indicative of his performance, that is, the smaller the mean square vertical velocity the better altitude control, generally speaking. Under these conditions, then, we can see from Figure 3 that the cutoff frequency gets lower for small values of  $\sigma_{va}$ . A low value of cutoff frequency implies that the filter is acting more like a true integrator, or, in other words, that the motion of the platform more accurately reproduces the motion of the aircraft.

The parameter  $w_1$  represents the lowest characteristic frequency of the system being simulated and is hence fixed by the particular problem at hand. The larger  $w_1$  is the higher the cutoff frequency will be, meaning that less of the actual motion of the system can be reproduced by the platform. Generally speaking, the frequency  $w_1$  will be larger for high performance than for low performance vehicles, so that we may expect more difficulties simulating the higher performance system.

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The parameter  $\overline{X_p}^2$  represents the limitation of the platform motion available. This parameter, being a statistic, has a direct influence on the tendency for the platform to try to exceed its limits of travel. If, as in the sample calculations plotted in Figure 3, the limit of the platform travel ( $\pm 3$  ft.) is used to set  $\overline{X_p}^2$  as 9 ft.<sup>2</sup> the probability that the platform will hit its limits is quite high (in the order of 0.3 assuming a Gaussian distribution). If some smaller probability is desirable or necessary it must be achieved at the cost of a reduction in fidelity of the simulated motion.

To summarize, the three parameters discussed must be considered for their combined effects on the simulation of the motion cues. Some compromise must be made of the fidelity of simulation if the subject is particularly poor in controlling the simulator, or if the aircraft is very responsive to control inputs, or if the probability of hitting the stops of the platform is to be kept small. In the final analysis adjustments in the filter response must be made on the basis of experience and hence the recommendations of this report are to be considered only as a starting point or as guide lines for experimentation.

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